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NUMERICAL ANALYSIS OF THE EFFECT OF LOCAL ENERGY SUPPLY ON THE AERODYNAMIC DRAG AND HEAT TRANSFER OF A SPHERICALLY BLUNTED BODY IN A SUPERSONIC AIR FLOW

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The effect of a local source of energy in a supersonic flow on the aerodynamic drag and heat transfer of a spherically blunted body is studied numerically. Calculations are performed on the basis of the Navier–Stokes equations for a thermally equilibrium model of air. Data on the effect of the intensity and size of the energy source on the wave drag, friction, and heat transfer are obtained. Particular attention is given to studying the effect of drag reduction by means of a focused heat source. The gas-dynamic nature of this effect is studied. The limits of drag reduction are estimated, and optimal conditions of heat supply are determined.

Introduction. Recently, there has been growing interest in various methods for controlled change in the flow structure and aerodynamic characteristics of flying vehicles by means of the remote action of a focused electromagnetic field (gas discharge). This problem was first formulated in Russia and has been studied in other countries. By now, the possibility of implementation of this idea into practice has been supported by laboratory experiments [1–4]. Chernyi [5, 6] studied numerically the interaction between a gas and an electromagnetic field and determined the gas-dynamic parameters of this process. Most gas-dynamic results were obtained with the use of the heat-source model according to which the absorption of electromagnetic energy is modeled by heat release with a specified intensity distributed over a finite region of the flow. Georgievskii and Levin [7] considered the linear formulation of the problem and showed that the energy supply at the segment in front of a narrow axisymmetric body is highly efficient for wave-drag reduction. These researches also showed [8] that the flow field can be drastically changed and the wave drag can be decreased by supplying a small amount of energy in a local zone upstream of the blunted body. The effect of local energy supply on the wave drag of axisymmetric sharp and blunt bodies of various configurations was studied in [9–12]. The occurrence of separation zones and considerable reduction (up to 50%) in the wave drag were observed. The energy saved was found to exceed manyfold the energy consumed.

Inviscid spatial flows with energy supply upstream of simple-shaped bodies were studied in [13–15]. The results obtained support the fact that the lift force and stalling torque can be changed by the energy supplied to the incoming flow.

Levin et al. [15, 16] calculated a supersonic flow of a viscous heat-conducting gas around a spherically blunted body. An analysis of these data shows that the drag can be significantly reduced even for rather low values of heat-supply intensity; in this case, the thermal load increases insignificantly.

In this paper, we give results of a numerical analysis of a supersonic axisymmetric air flow near the front part of a sphere in the presence of a heat source in the incoming flow. The calculations are performed

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on the basis of the Navier–Stokes equations for a thermally equilibrium model of air, which were integrated by the finite-volume method. The effect of local energy supply on the flow structure, aerodynamic forces, and heat transfer is studied as a function of the heat-supply intensity, heat-source size, and Mach and Reynolds numbers.

Gas-Phase Model. Air is considered as an ideal mixture of O_2 and N_2 with constant values of the molar concentrations X_m equal to 0.21 and 0.79, respectively. The rotational and vibrational degrees of freedom are described by the model "rigid rotator — harmonic oscillator" with characteristic vibrational temperatures $T_{\nu,O_2} = 2228$ K and $T_{\nu,N_2} = 3336$ K. In this gas-phase model, the gas state at the time-space point $(t, \boldsymbol{r})$ can be described by a set of independent variables $\boldsymbol{Z} = (p, \boldsymbol{u}, T)$, where p is the pressure, $\boldsymbol{u}$ is the velocity vector, and T is the temperature. The equation of state is written in the form

$$p = \rho R_a T/\bar{M},$$

where ρ is the density, R_a is the universal gas constant, and $\bar{M}$ is the average molecular weight of the mixture. The internal energy e per unit mass and the heat capacity for constant pressure c_p are given by

$$e = \frac{5}{2}\frac{R_a}{\bar{M}}T + \frac{R_a}{\bar{M}}\sum_m \frac{T_{\nu,m}X_m}{\exp(T_{\nu,m}/T) - 1},$$

$$c_p = \frac{7}{2}\frac{R_a}{\bar{M}} + \frac{R_a}{\bar{M}}\sum_m \frac{(T_{\nu,m}/T)^2 \exp(T_{\nu,m}/T) X_m}{(\exp(T_{\nu,m}/T) - 1)^2}.$$

The viscosity of the mixture is determined as a power function of temperature $\mu = a_\mu T^{0.683}$. The thermal conductivity of the mixture λ is determined from the condition that the Prandtl number is $\mathrm{Pr} = 0.7$.

Governing Equations. To calculate an axisymmetric gas flow, we use the Navier–Stokes equations for the above-described gas-phase model. In the cylindrical coordinates (x, y, φ), we write the governing equations in the integral form

$$\frac{d}{dt}\int_S \boldsymbol{U} y\, dS + \int_{\delta S} \boldsymbol{n}\cdot\boldsymbol{F} y\, dl = \int_S \boldsymbol{\Omega} y\, dS,$$

where S is a fixed control region in the meridional plane (x, y), δS is the boundary of this region, $\boldsymbol{n} = (n_x, n_y)$ is the outward unit normal to δS, $\boldsymbol{U}$ is the vector of conservative variables per unit volume, $\boldsymbol{F} = \boldsymbol{F}^{\rm inv} + \boldsymbol{F}^{\rm vis}$ is the sum of inviscid and viscid fluxes of the vector $\boldsymbol{U}$ through the boundary of the region, and $\boldsymbol{\Omega}$ is the vector of the source terms of the equations. For the above gas-phase model, these vectors are given by

$$\boldsymbol{F} = \left\{ \begin{array}{c} \rho\boldsymbol{u} \\ \rho u\boldsymbol{u} + p\boldsymbol{n}n_x \\ \rho v\boldsymbol{u} + p\boldsymbol{n}n_y \\ \rho\boldsymbol{u}h_0 \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ \boldsymbol{\tau}_x \\ \boldsymbol{\tau}_y \\ \boldsymbol{q} + u\boldsymbol{\tau}_x + v\boldsymbol{\tau}_y \end{array} \right\},$$

$$\boldsymbol{U} = \{\rho, \rho u, \rho v, \rho e_0\}^{\rm t}, \qquad \boldsymbol{\Omega} = \{0, 0, (p + \tau_{\varphi,\varphi})/y, \omega_h\}^{\rm t}.$$

Here u and v are the components of the vector velocity $\boldsymbol{u}$, $e_0 = e + 0.5(\boldsymbol{u}\cdot\boldsymbol{u})$ is the total energy per unit mass, $h_0 = e_0 + p/\rho$ is the total enthalpy, $\boldsymbol{\tau}_x = (\tau_{xx}, \tau_{xy})$ and $\boldsymbol{\tau}_y = (\tau_{yx}, \tau_{yy})$ are the viscous fluxes of momentum in the axial and radial directions, respectively, $\tau_{\varphi,\varphi}$ is the azimuthal component of the vector of the viscous flux of momentum in the axial direction, and $\boldsymbol{q}$ is the heat flux. The components of the vectors of viscous momentum fluxes correspond to nonzero components (taken with the opposite sign) of the viscous-stress tensor determined by the expression

$$\hat{\tau} = \mu\Big[\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{r}} + \Big(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{r}}\Big)^{\rm t} - \frac{2}{3}\Big(\frac{\partial}{\partial\boldsymbol{r}}\cdot\boldsymbol{u}\Big)\hat{I}\Big],$$

where $\hat{I}$ is the unit tensor. The heat flux $\boldsymbol{q}$ is set in the form

$$q = -\lambda \frac{\partial T}{\partial \boldsymbol{r}}.$$

It is assumed that the intensity of gas heating ω_h is distributed in space according to the Gauss law with the center located at the flow axis:

$$\omega_h = a_h \rho \exp\left[-(y^2 + (x - x_h)^2)/r_h^2\right].$$

Here a_h, x_h, and r_h are the prescribed parameters of the source.

Numerical Method. The equations are integrated by the finite-volume method on a curvilinear structured grid. In this approach, the system of finite-difference equations comprises numerical analogs of the laws of conservation of mass, momentum, and energy for quadrilateral cells that cover the calculation domain and a difference approximation of boundary conditions. The approximate solution $\boldsymbol{Z}_{i,j}$ is determined at the center of each cell $(x_{i,j}, y_{i,j})$ and at the center of each side of the cell $(x_{w,j}, y_{w,j})$ that lies on the body surface. The cells are formed by intersection of two discrete sets of curves. The inviscid fluxes $\boldsymbol{F}_G^{\text{inv}}$ through the cell boundaries are calculated from the exact solution of the Riemann problem $\boldsymbol{Z}_G = \mathcal{R}(\boldsymbol{Z}_G^L, \boldsymbol{Z}_G^R)$ ($\mathcal{R}$ is an operator of the solution of the problem). The left and right boundary values $\boldsymbol{Z}_G^L$ and $\boldsymbol{Z}_G^R$ inside the cells are calculated by limited one-dimensional extrapolation of the vector $\boldsymbol{Z}$ from the center of the cell toward the boundary. The numerical values of the viscous fluxes $\boldsymbol{F}_G^{\text{vis}}$ through the cell boundaries are calculated with the use of the second-order central and one-sided difference formulas. The difference equations are solved by a two-layered implicit iterative scheme based on the implicit approximation of unsteady Navier–Stokes equations. The implicit iterative operator is constructed with the use of the ($\pm$)-splitting of the Jacobi matrices of the numerical fluxes. Its approximate inversion is performed by the block variant of the successive relaxation method (Gauss–Seidel method) with LU-decomposition of block tridiagonal matrices.

Results of Calculation of a Flow around a Spherically Blunted Body in the Thermal Wake. First, we analyze the flow field structure near a sphere of radius R, which is located in the thermal far wake. As the determining parameters of the flow, we choose the following dimensionless quantities: free-stream Mach number M_∞, Reynolds number $\mathrm{Re}_{R,\infty}$ based on the free-stream parameters and the sphere radius R, vibrational temperatures $T_{\nu,\mathrm{O}_2}/T_\infty$, and $T_{\nu,\,\mathrm{N}_2}/T_\infty$, the ratio r_h/R, the ratio of the distance between the source center and stagnation point on the sphere to the radius of the sphere d/R, the temperature factor T_w/T_∞, where T_w is the surface temperature, and the parameter of heat-release intensity Q_H determined by the formula

$$Q_H = 2\pi \int \frac{\omega_h y}{\rho_\infty u_\infty h_\infty \pi r_h^2}\, dx\, dy$$

(integration is performed over the entire calculation domain). The parameter Q_H is the ratio of the total power of energy supply to the free-stream enthalpy flux through the characteristic section of the heat source.

Calculations were performed for certain combinations of Mach and Reynolds numbers ($\mathrm{M}_\infty = 1.5$, 3, and 6 and $\mathrm{Re}_{R,\infty} = 10^3$, 10^4, and 10^5) for $d/R = 7.5$–30 and dimensionless intensity of the heat source $Q_H = 0$–63. The temperature factor was assumed to be constant and equal to 1.2.

According to [15], the following three regimes of the flow around a sphere located in the heat-source wake can be distinguished: quasi-uniform, transitional, and abnormal regimes. Figure 1a–c shows the typical distributions of the pressure coefficient C_p, friction coefficient C_f, and Stanton number C_h multiplied by $\mathrm{Re}_{R,\infty}^{0.5}$ as functions of the distance s from the stagnation point on the sphere, respectively. These parameters were calculated by the formulas

$$C_p = p_w/(0.5\rho_\infty u_\infty^2), \quad C_f = \tau_w/(0.5\rho_\infty u_\infty^2), \quad C_h = q_w/(\rho_\infty u_\infty (h_{0,\infty} - h_{0,w})),$$

where p_w, τ_w, and q_w are the pressure, friction, and heat flux on the wall, respectively.

These results were obtained for $\mathrm{M}_\infty = 3$, $\mathrm{Re}_{R,\infty} = 10^4$, and $Q_H = 2$. Curves 1, 2, and 3 in Fig. 1 refer to the abnormal ($r_h/R = 0.05$, $d/r_h = 30$), transitional ($r_h/R = 0.35$, $d/r_h = 17$), and quasi-uniform ($r_h/R = 0.8$, $d/r_h = 7.5$) regimes, respectively. For comparison, Fig. 1 also shows these parameters obtained without heat supply (curves 4).

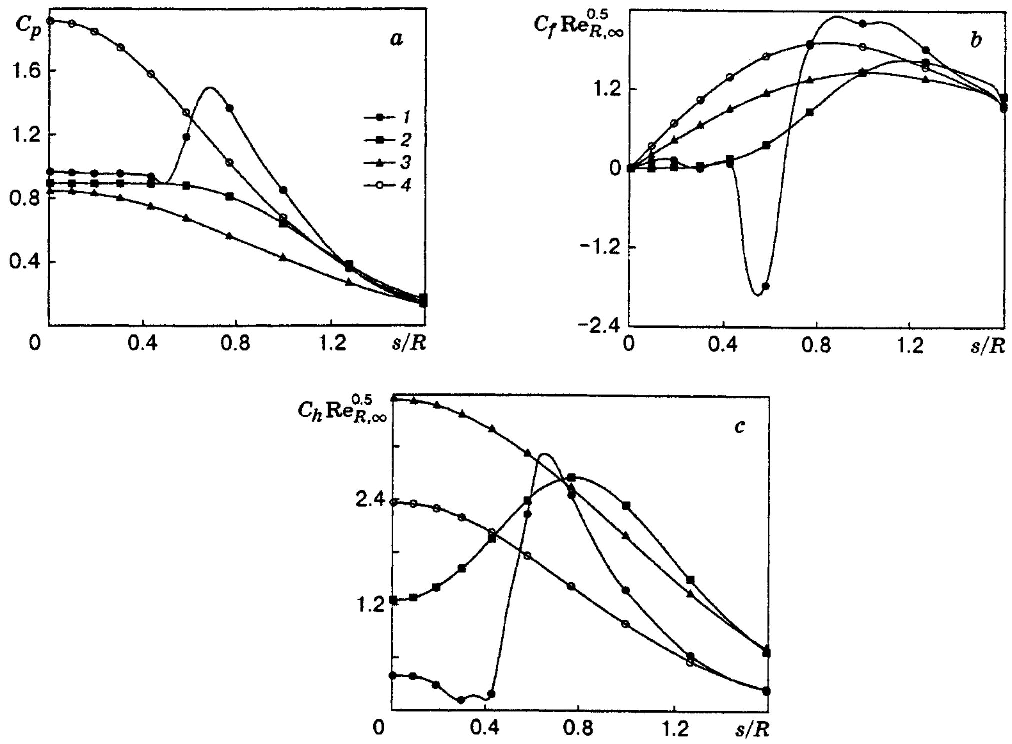

Fig. 1

The quasi-uniform regime occurs for $r_h/R \gtrsim 1$. In this case, the free stream with parameters close to the axial values in the thermal wake is almost uniform. Since the Mach number in the wake and the axial component of the momentum flux are smaller than their free-stream values, the distance from the bow shock wave to the body surface increases, and the wall pressure and the wave drag can be lower than those without heat supply. The maximum value and the total heat flux toward the body surface increase with increasing intensity of the heat source.

The transitional regime is observed if the flow upstream of the body is significantly nonuniform, whereas the flow in the shock layer remains attached [the quantity C_f remains positive (Fig. 1b)]. In this case, a stagnation zone with almost constant pressure and relatively high temperature of the gas is formed near the central part of the sphere surface. It has the shape of a blunted cone that smoothly joins the sphere surface. In this zone, the pressure distribution over the surface has a characteristic plateau with a reduced pressure that corresponds to the free-stream parameters near the axis (Fig. 1a). In the neighborhood of the boundary of this plateau, the heat flux reaches a maximum value determined by the density of the energy flux along the axis.

A further decrease in the ratio r_h/R leads to the appearance of a region of circulation flow with one or several vortices of different intensity in the shock layer, which is seen from the distribution of the friction coefficient (Fig. 1b). Levin et al. [15] called this regime the abnormal regime. Beginning from a certain minimum value of Q_H, a separation zone is formed and expands in the longitudinal and radial directions as the heat supply increases.

The developed separation zone is also shaped like a blunted cone, but it attaches to the sphere surface at a certain angle. Compression waves, which occur in the vicinity of the attachment zone, converge to a barrel shock wave. Interacting with the bow shock wave, the barrel shock deflects it toward the flow. This leads to the formation of a centered rarefaction wave and tangential discontinuity. For sufficiently large angles of deflection, a local subsonic zone occurs behind the bow shock wave.

A distinctive feature of the surface-pressure distribution for the abnormal regime is the occurrence of a peripheral maximum that initiates the main circulation flow (Fig. 1a). A local peak of the wall pressure is located near the boundary of the circulation zone. Inside this zone, the pressure is reduced and close to the value of pressure in the transition regime for the same heat-supply parameter. The peripheral maximum compensates partly or completely (for small Q_H) the pressure decrease in the central part of the body surface. Therefore, the decrease in the drag due to the heat supply becomes pronounced for relatively large values of Q_H when the separation zone is sufficiently developed. In the region of detached flow, there is a relatively cold gas; therefore, for the abnormal regime, the intensity of heat transfer in the central part of the sphere surface can be much smaller than it would be without heat supply. At the site of flow reattachment, the intensity of heat transfer increases, and for large Q_H, the heat flux here can exceed the maximum value attainable without energy supply (Fig. 1c).

The values of r_h/R, for which the flow regimes change depend on the similarity parameters, which affect the structure of the thermal wake. They decrease with an increase in Q_H and decrease in $\mathrm{Re}_{R,\infty}$.

Aerodynamic Drag and Heat Transfer. Figures 2–4 show the drag coefficient C_x and the maximum Stanton number over the surface $C_{h,\max}$ calculated by the formulas

$$C_x = C_{px} + C_{fx}, \quad C_{h,\max} = \max_s C_h(s),$$

$$C_{px} = -2\pi \int_0^{\pi R/2} \frac{p_w n_x y}{0.5\rho_\infty u_\infty^2 \pi R^2}\,dl, \quad C_{fx} = -2\pi \int_0^{\pi R/2} \frac{(\boldsymbol{\tau}_{xw}\cdot \boldsymbol{n})y}{0.5\rho_\infty u_\infty^2 \pi R^2}\,dl.$$

These quantities are normalized to their values without heat supply and considered as functions of the dimensionless intensity of the heat source Q_S. The parameter Q_S is proportional to the ratio of the total power of heat supply to the kinetic-energy flux of the incoming flow through the midsection of an aerodynamic body:

$$Q_S = 2\pi \int \frac{\omega_h y}{0.5C_x(0)\rho_\infty u_\infty^3 \pi R^2}\,dx\,dy.$$

Here $C_x(0)$ is the drag coefficient for the case where no energy is supplied. The heat-exchange parameters Q_S and Q_H are related (for $T_\infty \ll T_{\nu,m}$) by the formula

$$Q_S = \frac{2}{C_x(0)(\gamma_\infty - 1)\mathrm{M}_\infty^2}\,\frac{r_h^2}{R^2}\,Q_H,$$

where γ is the ratio of specific heats. This representation of results allows one to estimate the efficiency E of the heat contribution to drag reduction, which is determined as the ratio of the conserved energy to the energy consumed to heat the gas:

$$E = \frac{(C_x(0) - C_x(Q_S))0.5\rho_\infty u_\infty^3 \pi R^2}{0.5Q_S C_x(0)\rho_\infty u_\infty^3 \pi R^2} = \frac{1 - \bar{C}_x(Q_S)}{Q_S},$$

where $\bar{C}_x(Q_S) = C_x(Q_S)/C_x(0)$. The distributions of this parameter are shown in Figs. 2a–4a by dashed curves.

Figure 2 shows the effect of the heat-source size on drag reduction and heat transfer for $\mathrm{M}_\infty = 3$, $\mathrm{Re}_{R,\infty} = 10^4$, and $Q_S = 0$–1. The data corresponding to $r_h/R = 0.05$ (curves 1) are obtained for $d/r_h = 30$ and those corresponding to $r_h/R = 0.1$, 0.2, and 0.4 (curves 2–4) are obtained for $d/r_h = 15$. One can see from Fig. 2a that, as r_h/R decreases, the efficiency of heat supply required to reduce the drag to a specified value increases. The minimum value of drag calculated for each size of the source corresponds to the maximum value of the heat parameter $\bar{Q}_H = 63$ for all r_h/R. The minimum value of $C_x(Q_S)/C_x(0)$ decreases as r_h/R increases, the efficiency of heat supply decreasing significantly. An analysis shows that the maximum reduction in drag can be obtained if a heat source of size of the order of the sphere radius R is used and the intensity parameter Q_H is sufficiently large. The limiting values of $C_x(Q_S)/C_x(0)$ decrease

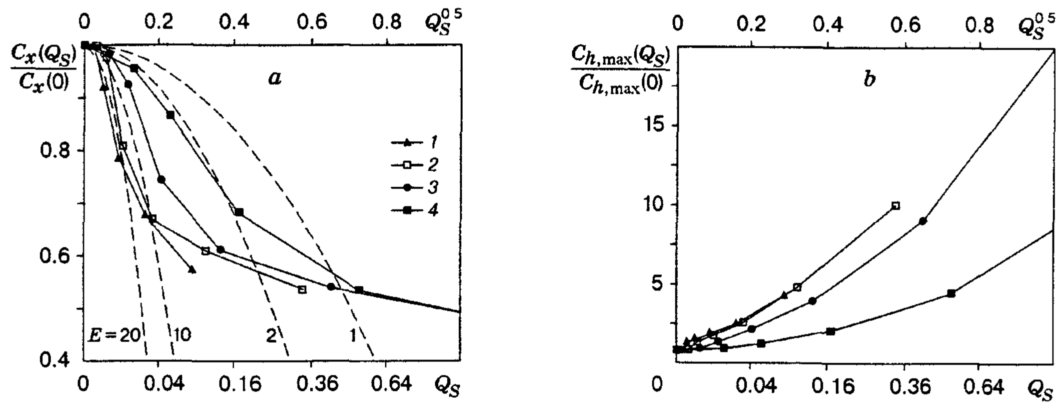

Fig. 2

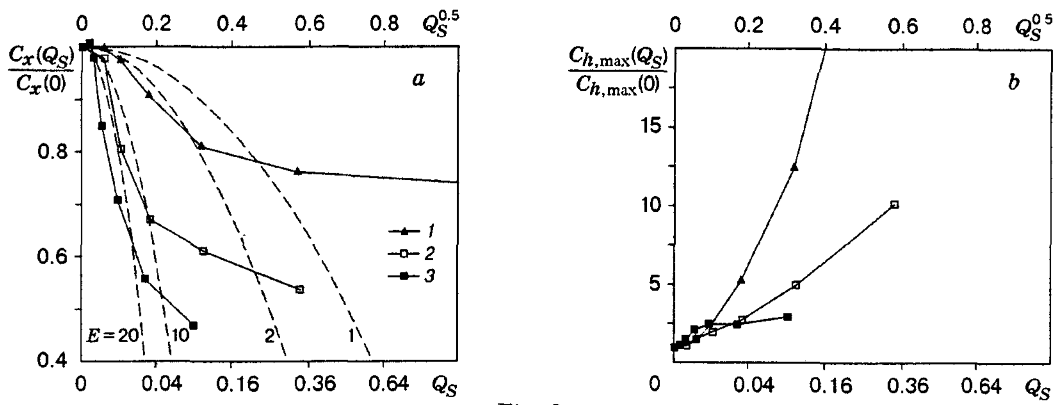

Fig. 3

from 0.68 for $M_\infty = 1.5$ to 0.32 for $M_\infty = 6$. The calculation results show that realization of these regimes requires a contribution of energy that exceeds manyfold the energy required to overcome the drag (without energy supply). Moreover, under these conditions, the intensity of heat exchange with the surface increases tenfold or more.

From Fig. 2a, one can see that, for a relatively small size of the heat source, there is a tendency for further decrease in aerodynamic drag with an increase in the heat-supply parameter for $Q_H > \bar{Q}_H$. Probably, this is attributed to the fact that the wake radius increases with an increase in Q_H.

Figure 2b shows the effect of the source size on the dependence $C_{h,\max}(Q_S)$. The data presented show that the maximum heat flux toward the surface increases as Q_S increases; however, the character of this dependence differs from that of the dependence $C_x(Q_S)$. As a result, a considerable reduction in drag can be obtained for relatively small (twofold or threefold) increase in the intensity of surface heating.

Figure 3 shows the effect of the Mach number on drag reduction and heat transfer (curves 1–3 refer to $M_\infty = 1.5$, 3, and 6, respectively). The data were obtained for $r_h/R = 0.1$, $\mathrm{Re}_{R,\infty} = 10^4$, and $d/r_h = 15$. One can see that the efficiency of heat supply used for drag reduction increases with increase in M_∞ (Fig. 3a). For example, the efficiency of the use of heat supply to reduce the drag by 20% increases by more than ten times as the Mach number increases from 1.5 to 6. At the same time, as is seen in Fig. 3b, the relative increase in the parameter $C_{h,\max}(Q_S)/C_{h,\max}(0)$ caused by energy supply becomes much less intense with increasing Mach number, except for the region of low values of heat-supply intensity ($Q_S < 0.02$).

Figure 4 shows the dependences $C_x(Q_S)$ and $C_{h,\max}(Q_S)$ calculated for $\mathrm{Re}_{R,\infty} = 10^3$, 10^4, and 10^5 (curves 1–3, respectively), $M_\infty = 3$, $r_h/R = 0.1$, and $d/r_h = 15$. One can see from Fig. 4a that the effect of viscosity of the gas on drag reduction by means of energy delivered to the incoming flow becomes appreciable for $\mathrm{Re}_{R,\infty} \leqslant 10^4$. With a decrease in the Reynolds number, the use of heat supply for drag reduction becomes

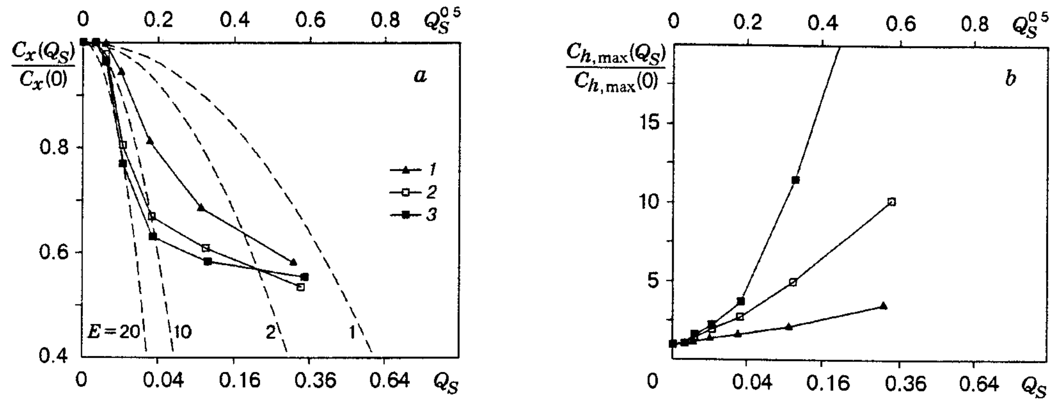

Fig. 4

less efficient as a whole. The effect of the Reynolds number on surface heating is very pronounced (Fig. 4b). The greater the value of $\mathrm{Re}_{R,\infty}$, the faster the maximum heat fluxes toward the sphere surface increase as the heat supplied to the incoming flow increases.

Conclusions. A parametric study of a supersonic flow around a spherically blunted body in the presence of an energy-supply source has been performed. Calculations have been carried out on the basis of the Navier–Stokes equations for the model of thermally equilibrium air for a wide range of free-stream parameters and the intensity and size of the heat source. The effect of heat supply on the flow field, aerodynamic drag, and heating of the body surface has been studied.

It is shown that the heat supplied to the incoming flow leads to a considerable reduction in the aerodynamic drag. Maximum reduction in the drag can be obtained if an intense heat source with characteristic size of the order of the sphere radius is used. The limiting values of drag reduction decrease from 0.68 for $\mathrm{M}_\infty = 1.5$ to 0.32 for $\mathrm{M}_\infty = 6$. However, the power of the heat source necessary to reach these limiting values exceeds considerably the power of an engine required to overcome the aerodynamic drag when moving in air without heat supply.

The efficiency of the use of heat supply for drag reduction increases as the relative size of the heat source decreases and the Mach and Reynolds numbers increase. For example, for $\mathrm{M}_\infty = 3$, $\mathrm{Re}_{R,\infty} = 10^3$, and $r_h/R = 0.1$, the drag of a hemisphere can be reduced by 20% for an efficiency of the heat supply greater than 2000% ($E > 20$).

Heat supply intensifies heat exchange, but owing to the different character of the dependences of the drag and heating on the heat-supply intensity, it is possible to obtain a significant reduction in drag for a relatively small increase in thermal loads on the surface of the aerodynamic body.

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